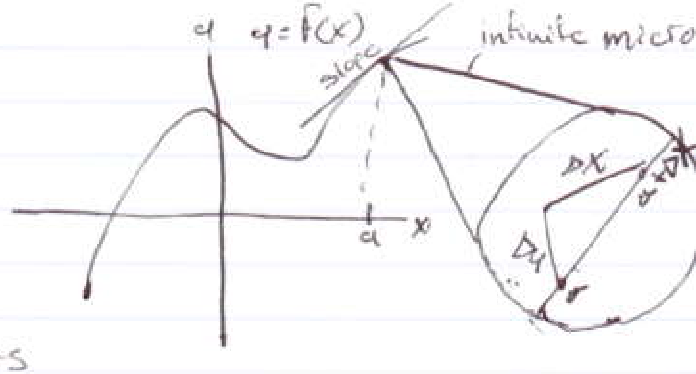
(Notes) Calcolas

2.1

Slope of a Curve

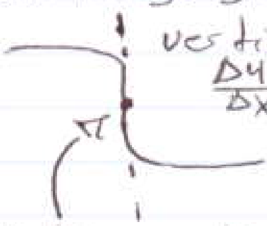


The slope at $a=x$ of $f(x)$
is $st\left(\frac{f(a+\Delta x)-f(a)}{\Delta x}\right)$

provided this nomber exsists
and is the same for all
nonzero infinitesimal Δx.

$\Delta x = (a+\Delta x) - a$
$\Delta y = f(a+\Delta x) - f(a)$
$\text{slope} = \frac{\Delta y}{\Delta x}$

How things go wrong:



vertical tangent line
$\frac{\Delta y}{\Delta x}$ infinite

Curve has no slope here

$st\left(\frac{\Delta y}{\Delta x}\right)$ changes if you change Δx

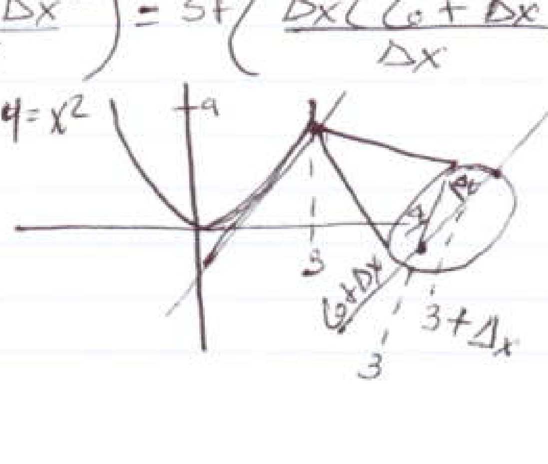
Ex. $y = f(x) = x^2$
Find slope at $x=3$
Let $x\Delta \neq 0$ and $\Delta x \approx 0$.

$$\text{slope} = st\left(\frac{(3+\Delta x)^2 - 3^2}{\Delta x}\right) = st\left(\frac{9+6\Delta x+\Delta x^2-9}{\Delta x}\right)$$

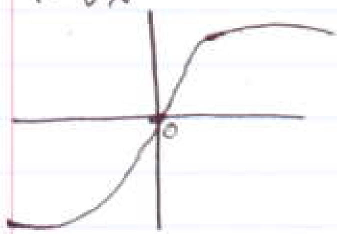
$$\left((a+b)^2 = a^2+2ab+b^2\right)$$

$$= st\left(\frac{6\Delta x+\Delta x^2}{\Delta x}\right) = st\left(\frac{\Delta x(6+\Delta x)}{\Delta x}\right) = st\,(6+\Delta x)$$

$$= 6+0 = 6$$



$y = \sqrt[3]{x}$



try to find slope at $x=0$

$$st\left(\frac{\sqrt[3]{0+\Delta x} - \sqrt[3]{0}}{\Delta x}\right) = st\left(\frac{\sqrt[3]{\Delta x}}{\Delta x}\right)$$

Δx is a nonzero infinitesimal

$$st\left(\frac{\Delta x^{1/3}}{\Delta x^{3/3}}\right) = st\left(\frac{1}{\Delta x^{2/3}}\right) = st\left(\frac{1}{\sqrt[3]{\Delta x^2}}\right)$$

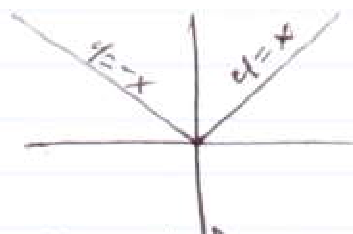
$\Delta x \approx 0$

$\Delta x^2 \approx 0$ infinite $\Rightarrow st\left(\frac{1}{\sqrt[3]{\Delta x^2}}\right)$

$\sqrt[3]{\Delta x^2} \approx 0 \Rightarrow \frac{1}{\sqrt[3]{\Delta x^2}}$

Since this DNE, $\sqrt[3]{x}$ has no slope at $x=0$.

$$y = F(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



Try to find slope at $x=0$

$$st\left(\frac{|0+\Delta x| - |0|}{\Delta x}\right) = st\left(\frac{|\Delta x|}{\Delta x}\right)$$

$(\Delta x \neq 0 \text{ and } \Delta x \approx 0)$

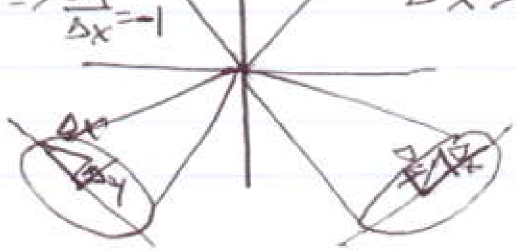
if $x > 0$

$$st\left(\frac{\Delta x}{\Delta x}\right) = st(1) = 1$$

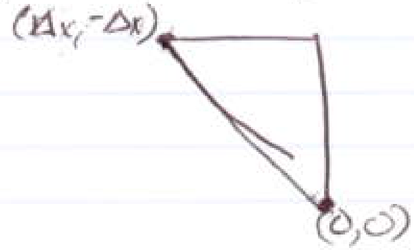
if $\Delta x < 0$ $st\left(\frac{-\Delta x}{\Delta x}\right) = st(-1) = -1$

conclude $y = |x|$ has no slope at $x=0$.

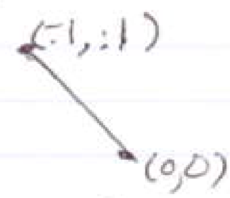
$\Delta x < 0 \Rightarrow \frac{\Delta y}{\Delta x} = -1$ $\Delta x > 0 \Rightarrow \frac{\Delta y}{\Delta x} = 1$



Try to find slope at $x=0$:



slope $= \dfrac{-\Delta x - 0}{\Delta x - 0}$ and slope $= \dfrac{0-(-\Delta x)}{0-\Delta x}$

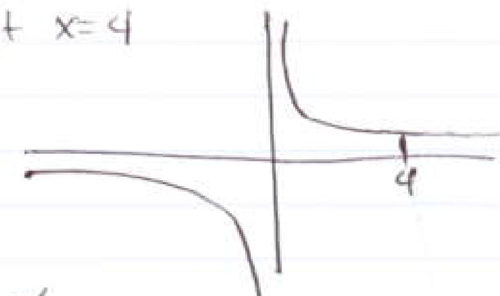


$\begin{cases} \Delta x = 0-(-.1) \\ \Delta y = 0 - .1 \end{cases}$

or

$\begin{cases} \Delta x = -.1 - 0 \\ \Delta y = .1 - 0 \end{cases}$

Find slope of $\frac{1}{x} = f(x)$
at $x = 4$



+ slope → small slope →

− slope → big slope

$$st\left(\frac{\frac{1}{4+\Delta x} - \frac{1}{4}}{\Delta x}\right) = st\left(\frac{\frac{1}{4+\Delta x}\cdot\frac{4}{4} - \frac{1}{4}\cdot\frac{4+\Delta x}{4+\Delta x}}{\Delta x}\right)$$

$$st\left(\frac{\frac{4}{16+4\Delta x} - \frac{4+\Delta x}{16+4\Delta x}}{\Delta x}\right) = st\left(\frac{\frac{-\Delta x}{16+4\Delta x}}{\Delta x}\right) = st\left(\frac{-\Delta x}{\Delta x(16+4\Delta x)}\right)$$

$$= st\left(\frac{-1}{16+4\Delta x}\right) = \text{slope} = \frac{-1}{16} = \frac{-1}{16+4\cdot 0}$$