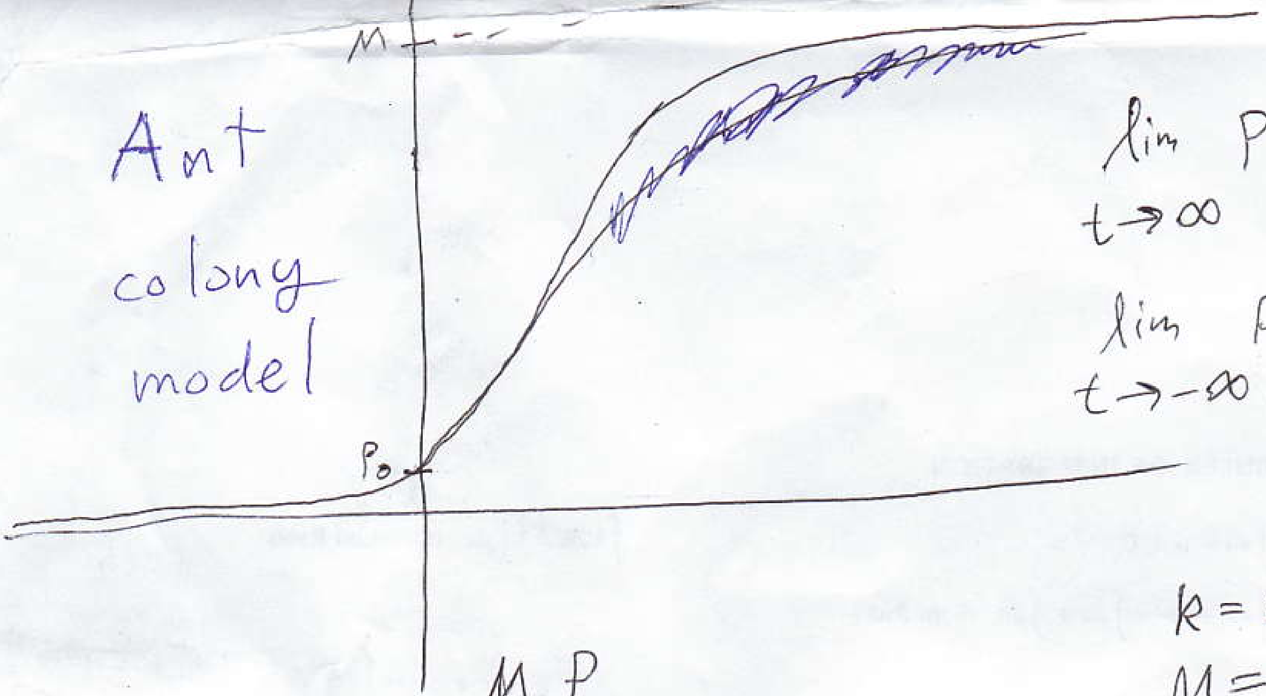
Ant colony model



$$\lim_{t\to\infty} P(t) = M$$

$$\lim_{t\to-\infty} P(t) = 0$$

$k = \frac{1}{\text{years}}$

$M = 10^4$ ants

$$P(t) = \frac{M P_0}{P_0 + (M - P_0)e^{-kt}}$$

$P(0) = P_0 = 10^3$ ants

$$\frac{1}{3\text{yrs}}\int_0^{3\text{ years}} P(t)\,dt =$$ average population ~~over~~ from $t=0$ years to $t=3$ years ~~1st 3 years~~

t	$P(t)$	t	$P(t)$
−4	20	4	8,585
−3	55	5	9,428
−2	148	6	9,782
−1	393	7	9919
0	1,000	8	9970
1	2,320	9	9989
2	4,509	10	9996
3	6,906		

Marco Lopez

$$\lim_{t\to\infty} P(t) = st\left(\frac{MP_0}{P_0+(M-P_0)e^{-kH}}\right) = \frac{MP_0}{P_0} = M$$

H positive infinite
$-kH$ negative infinite
$e^{kH} \simeq 0$

$$\lim_{t\to-\infty} P(t) = st\left(\frac{MP_0}{P_0+(M-P_0)e^{-k(-H)}}\right) = 0$$

$e^{-k(-H)} = e^{kH}$ H positive infinite
kH positive infinite
e^{kH} positive infinite
$\underbrace{(M-P_0)}_{good} e^{kH}$ positive infinite
$P_0+(M-P_0)e^{kH}$ positive infinite

$$\frac{MP_0}{P_0+(M-P_0)e^{kH}} = \frac{finite}{infinite} = infinitesimal$$

Average population from $t=0$ to $t=3$ years:

$$avg. = \frac{1}{3-0}\int_0^3 P(t)\,dt = \frac{1}{3(years)}\int_0^{3(years)} \frac{MP_0\,dt}{P_0+(M-P_0)e^{-kt}}\cdot\frac{e^{kt}}{e^{kt}}$$

$$= \frac{1}{3yrs}\int_0^{3yrs}\frac{MP_0e^{kt}\,dt}{P_0e^{kt}+M-P_0}$$

$u = P_0e^{kt}+M-P_0$
$du = (P_0e^{kt}+M-P_0)'\,dt$
$du = P_0e^{kt}(kt)'+0-0)\,dt$
$du = P_0e^{kt}k\,dt$
$\frac{M}{k}\,du = MP_0e^{kt}\,dt$

$$= \frac{1}{3yrs}\int_M^{P_0(e^3-1)+M}\frac{(M/k)\,du}{u}$$

$$\text{average} = \frac{1}{3yrs}\cdot\frac{M}{k}\int_M^{P_0(e^3-1)+M}\frac{du}{u}$$

$k = \frac{1}{year}$

$t=0 \Rightarrow u = P_0e^0+M-P_0$
$= P_0+M-P_0 = M$
$t = 3years \Rightarrow u = P_0e^{k(3years)}+M-P_0$
$u = P_0(e^3-1)+M$

$= \frac{M}{3} \ln|u| \Big|_{M}^{P_0(e^3-1)+M}$ $\quad u>0 \Rightarrow |u| = u$

$P_0 = 1{,}000$ ants
$M = 10{,}000$ ants

$= \frac{M}{3}(\ln(P_0(e^3-1)+M) - \ln M)$

$= \frac{M}{3}\ln\left(\frac{P_0(e^3-1)+M}{M}\right) = 3{,}558.85$ ants

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Limit with hyperreals

$\lim_{x\to 2}(3x+5) = 11$

means for all ε where $0 \simeq \varepsilon \neq 0$, we have
$3(2+\varepsilon)+5 \simeq 11$ (Same as $st(3(2+\varepsilon)+5) = 11$).

Easy to prove that
$\lim_{x\to 2}(3x+5) = 11$

Let $0 \neq \varepsilon \simeq 0$
$3(2+\varepsilon)+5 \simeq 3(2+0)+5 = 3\cdot 2+5 = 11$ ✓

Limits without hyperreals

$\lim_{x\to 2}(3x+5) = 11$ means

there is a function $\delta(\varepsilon)$ such that for all $\varepsilon > 0$,
we have $\delta(\varepsilon) > 0$ and $[2-\delta < x < 2+\delta \;\&\; x \neq 2] \Rightarrow$
$(11-\varepsilon < 3x+5 < 11+\varepsilon)$

For example, $\delta = \frac{\varepsilon}{3}$ works.

$\Updownarrow$

$\frac{11}{3} + \frac{\varepsilon}{3} < x + \frac{5}{3} < 11 + \frac{\varepsilon}{3}$

$\Updownarrow$

$2 - \frac{\varepsilon}{3} < x < 2 + \frac{\varepsilon}{3}$

$$\lim_{x\to\infty} \frac{2x}{x+5} = 2$$

means if H positive infinite,
then $\frac{2H}{H+5} \approx 2$ (Same as $st\left(\frac{2H}{H+5}\right) = 2$)

Easy to prove that
$$\lim_{x\to\infty} \frac{2x}{x+5} = 2$$

$$\frac{2H}{H+5} = \frac{2H/H}{(H+5)/H} = \frac{2}{1+5/H} \approx \frac{2}{1+0} = \boxed{2}$$

Limits without hyperreals
$\lim_{x\to\infty} \frac{2x}{x+5} = 2$ means

There is a function $M(\varepsilon)$ such that for all $\varepsilon > 0$,
$$x > M(\varepsilon) \Rightarrow \underbrace{2-\varepsilon < \frac{2x}{x+5} < 2+\varepsilon}$$

$$\Updownarrow$$

$$\frac{1}{2-\varepsilon} > \frac{x+5}{2x} > \frac{1}{2+\varepsilon}$$

$$\Updownarrow$$

$$\frac{1}{2-\varepsilon} > \frac{1}{2} + \frac{5}{2x} > \frac{1}{2+\varepsilon}$$

$$\Updownarrow$$

$$\frac{1}{2-\varepsilon} - \frac{1}{2} > \frac{5}{2x} > \overbrace{\frac{1}{2+\varepsilon} - \frac{1}{2}}^{\text{negative}}$$

$$\Uparrow$$

$$\frac{1}{\frac{1}{2-\varepsilon} - \frac{1}{2}} < \frac{2x}{5} \overbrace{> 0}^{\text{redundant}}$$

$$\Updownarrow$$

$$\frac{5}{\frac{2}{2-\varepsilon} - 1} < x \qquad M(\varepsilon) = \frac{5}{\frac{2}{2-\varepsilon} - 1} \text{ works}$$

For example $M\left(\frac{1}{10}\right) = 95$

$\left[x > 95 \Rightarrow 1.9 < \frac{2x}{x+5} < 2.1\right]$

Compare with hyperreals version:
x positive infinite $\Rightarrow \frac{2x}{x+5} \approx 2$