Test 2: Average: 71

Median: 79

σ : 24

Work (physics)

Work = force × distance

force: often just weight in pounds (lbs)

distance: distance in direction of force, often gravity, so up/down

Lifting

a 10lb bag up 2ft does 20 ft·lbs of work.

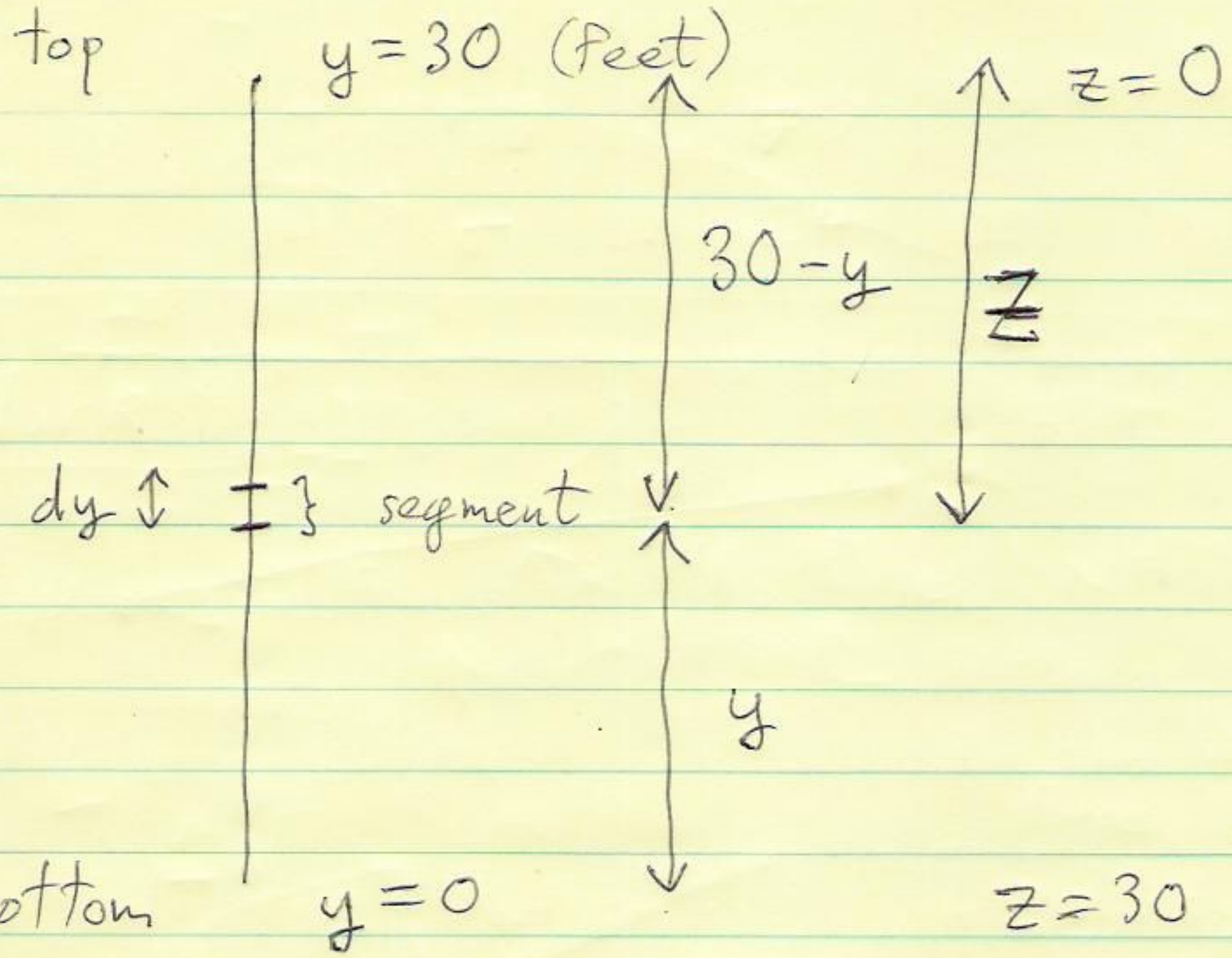
Integrals come in from "adding up" lots of little objects into an extended object.

~~rope has~~

~~linear density~~

rope weighs 15 lbs

How much work to lift the whole rope to the top?



$dW = \text{Work}_{\text{segment}} = \underbrace{\text{weight}_{\text{segment}}}_{dF} \cdot \underbrace{\text{distance}}_{(30-y)}$

$dF = ?$ Assume rope has a uniform density.

$\text{linear density} = \frac{\text{weight}}{\wedge} = \frac{15 \text{ lbs}}{30 \text{ ft}} = \frac{1}{2} \frac{\text{lb}}{\text{ft}}$

$\frac{dF}{dy} = \frac{1}{2} \frac{\text{lb}}{\text{ft}} \Rightarrow dF = \left(\frac{1}{2} \frac{\text{lb}}{\text{ft}}\right) dy$

$dW = \left(\frac{1}{2}\, dy\right)(30 - y)$

Work $W = \int_{y=0}^{y=30} \left(\frac{1}{2}\,dy\right)(30-y)$

$z = 30 - y$ $\quad dz = 0 - dy = -dy$

$dz = (0-1)dy$

$-dz = dy$

$y = 0 \Rightarrow z = 30 - 0 = 30$

$y = 30 \Rightarrow z = 30 - 30 = 0$

$$W = \int_{z=30}^{z=0} \left(\frac{1}{2}(-dz)\right)(z)$$

$$= \int_{z=0}^{z=30} \left(\frac{1}{2}\,dz\right) z$$

$$\int_a^b -f(x)\,dx = \int_b^a f(x)\,dx$$

$$= \frac{1}{2}\int_0^{30} z\,dz = \frac{1}{2}\left(\frac{z^2}{2}\right)\Big|_0^{30}$$

$$= \frac{1}{2}\left(\frac{30^2}{2} - \frac{0^2}{2}\right) = \frac{30^2}{4} = 15^2 =$$

$$= \boxed{225 \text{ ft} \cdot \text{lbs}}$$

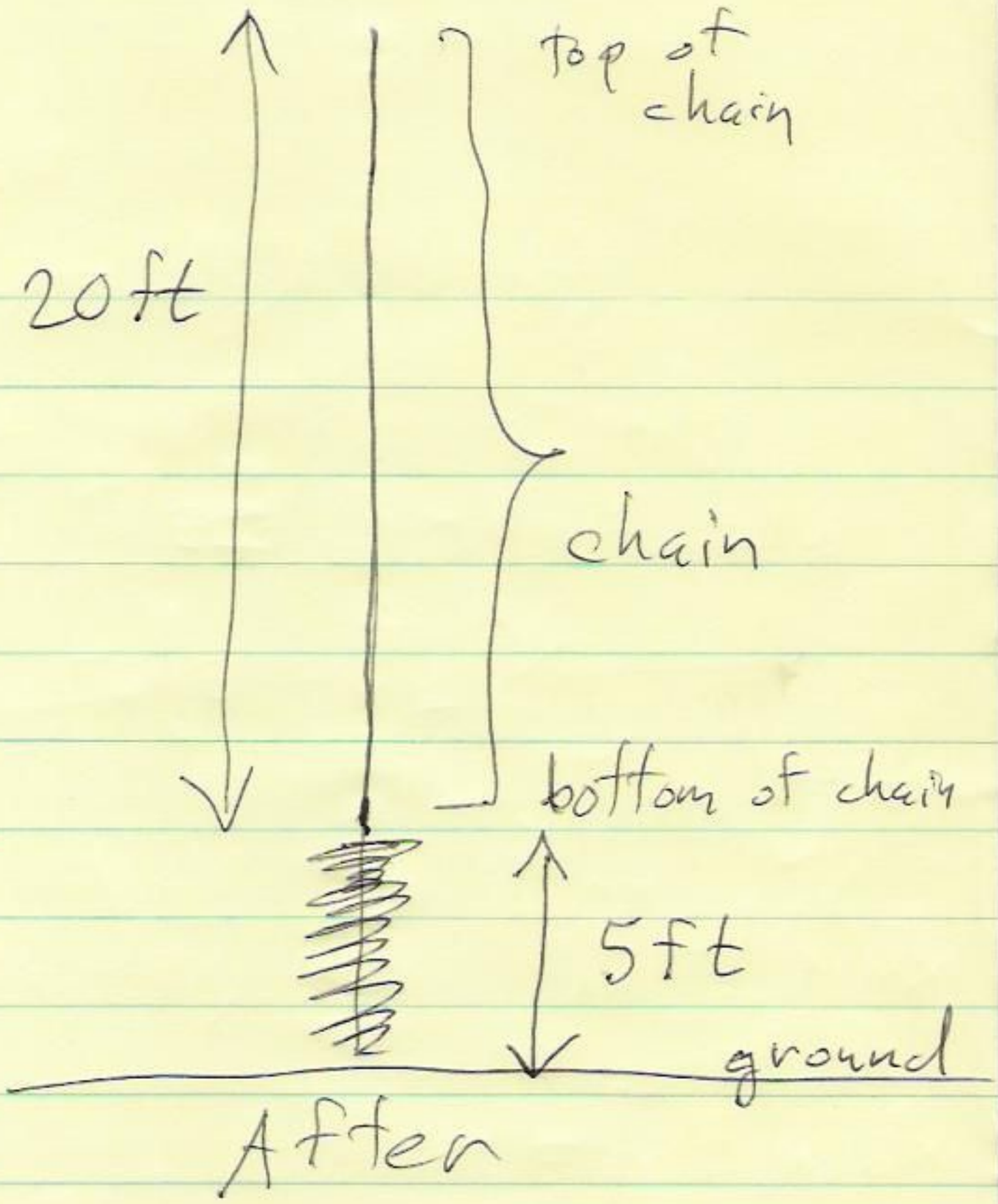
Alternative: $\int_0^{30} \left(\frac{1}{2}\,dy\right)(30-y) = \int_0^{30} \left(15 - \frac{y}{2}\right)dy$

$$= \left(15y - \frac{y^2}{4}\right)\Big|_0^{30} = \left(15 \cdot 30 - \frac{30^3}{4}\right) - (0 - 0)$$

Chain weighs
50 lbs

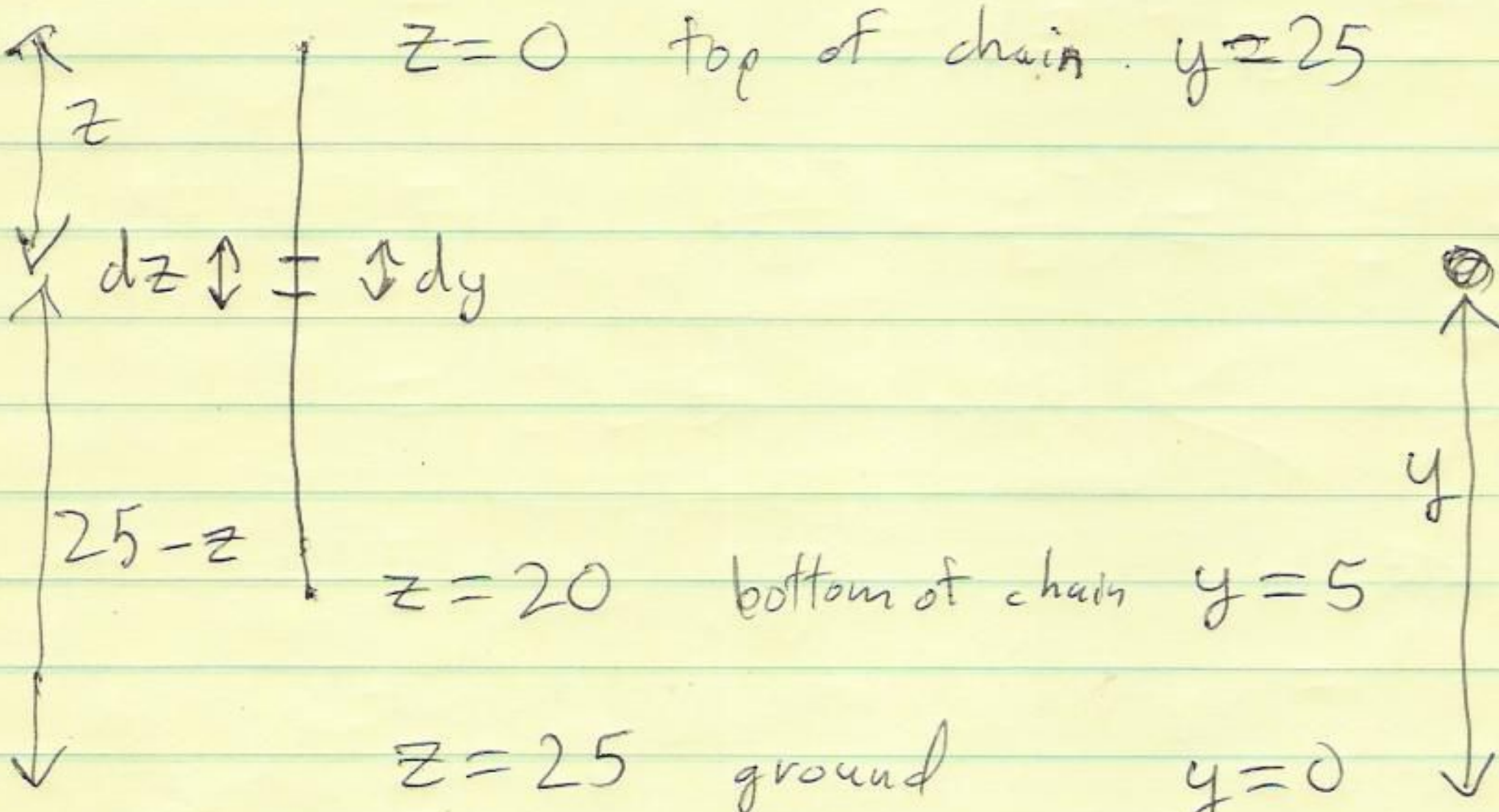
Before

chain on ground



After

How much work done?



$z = 0$ top of chain. $y = 25$

$z = 20$ bottom of chain $y = 5$

$z = 25$ ground $y = 0$

$$dW = dF \cdot y \qquad \frac{dF}{dy} = \frac{50\ lbs}{20\ ft} = 2.5$$

$$dF = 2.5\,dy$$

assuming constant density

$$dW = 2.5\,dy \cdot y = 2.5\,y\,dy$$

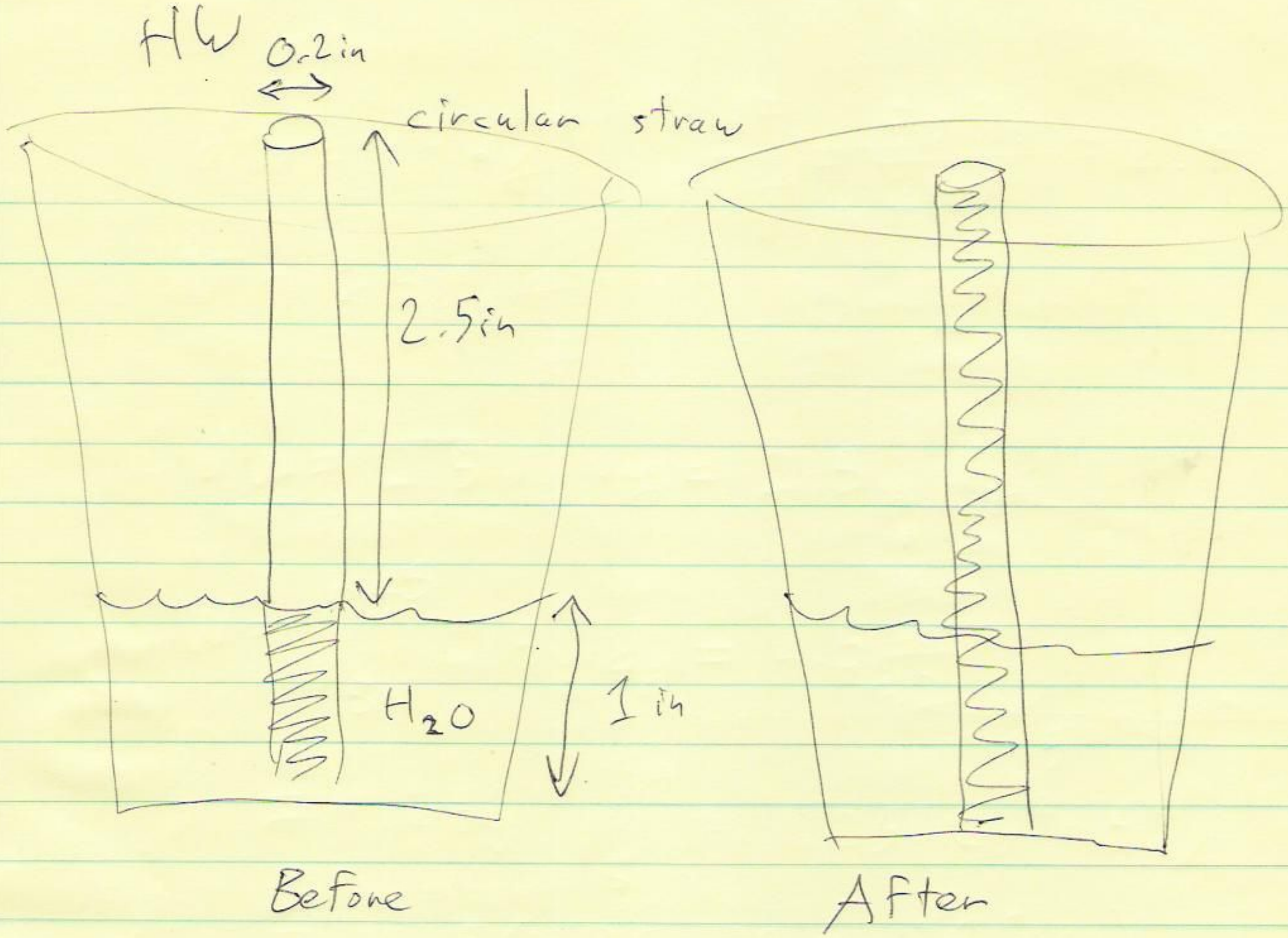
$$W = \int_5^{25} 2.5\,y\,dy = 2.5\left(\frac{y^2}{2}\right)\Big|_5^{25}$$

$$= 1.25y^2\Big|_5^{25} = 1.25(25^2 - 5^2)$$

$$= 1.25(625 - 25) = 1.25(600)$$

$$= \boxed{750\ ft \cdot lbs}$$

HW



Before After

Find work done