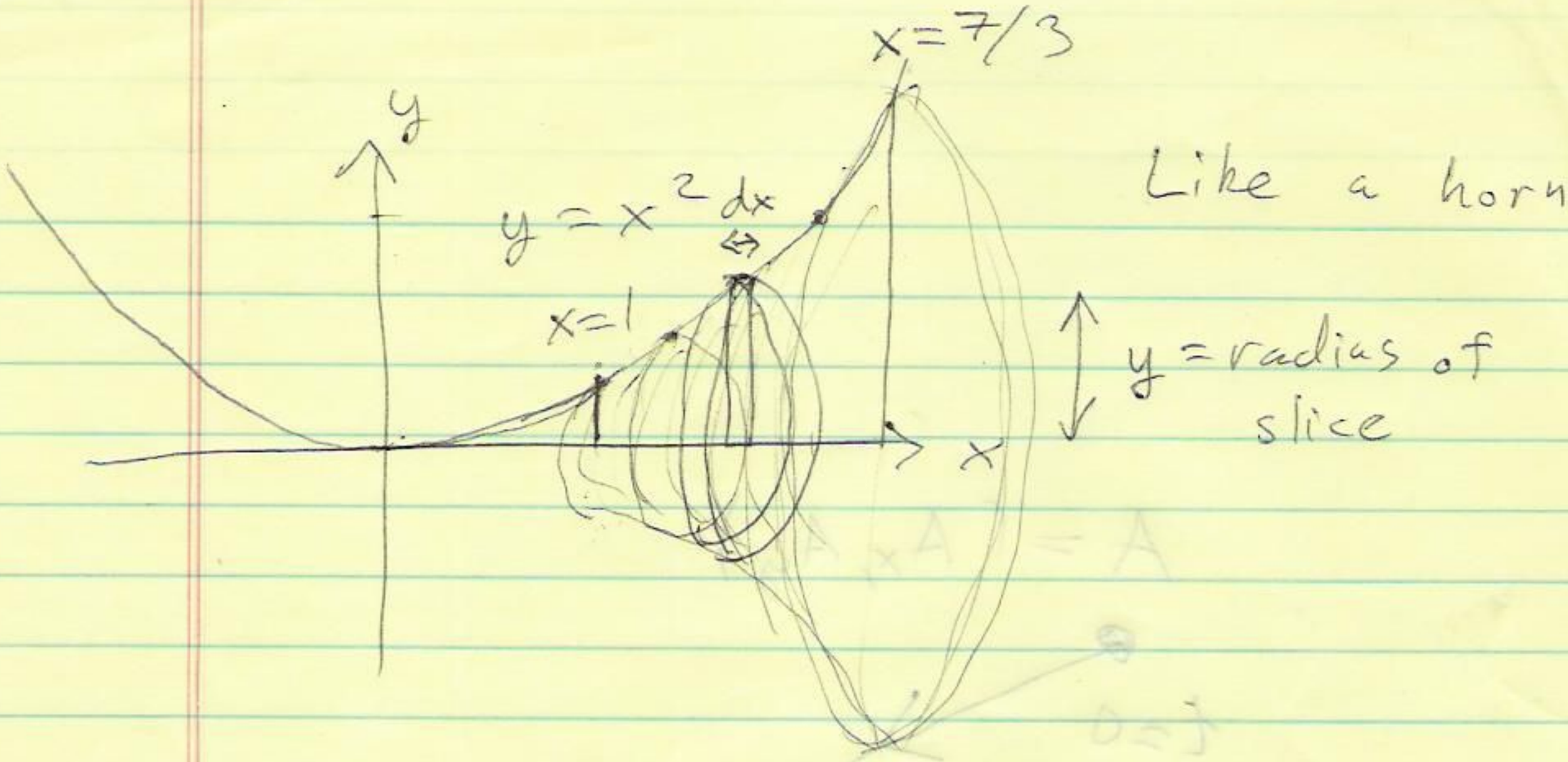
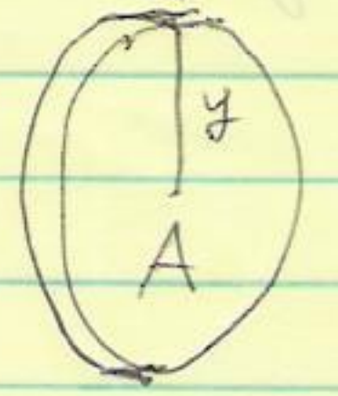


Revolve. $y=x^2$ from $x=1$ to $x=\frac{7}{3}$

around x-axis. Volume = ?

Slice $\leftrightarrow dx$ into disks;



$$dV = A\,dx = \pi y^2\,dx$$

$$V = \int_{x=1}^{x=7/3} \pi y^2\,dx$$

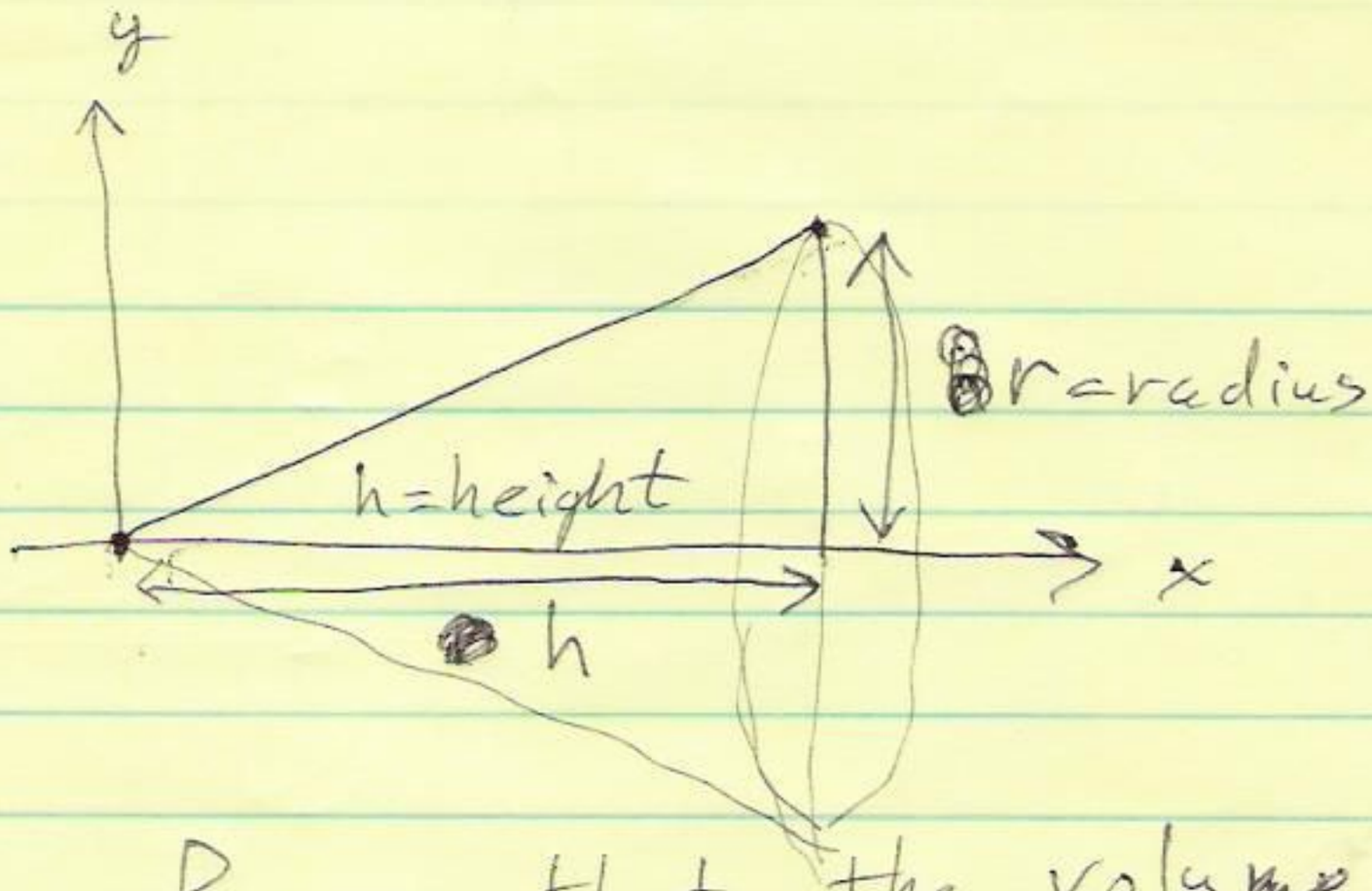
$$V = \int_{x=1}^{x=7/3} \pi (x^2)^2\,dx = \int_1^{7/3} \pi x^4\,dx$$

$$= \frac{\pi x^5}{5}\Big|_1^{7/3} = \frac{\pi}{5}\left(\left(\frac{7}{3}\right)^5 - 1^5\right)$$

Next Test - Wednesday (23rd)

calculator & notes OK

HW #1



Prove that the volume of a cone is $\frac{1}{3}\pi r^2 h$.

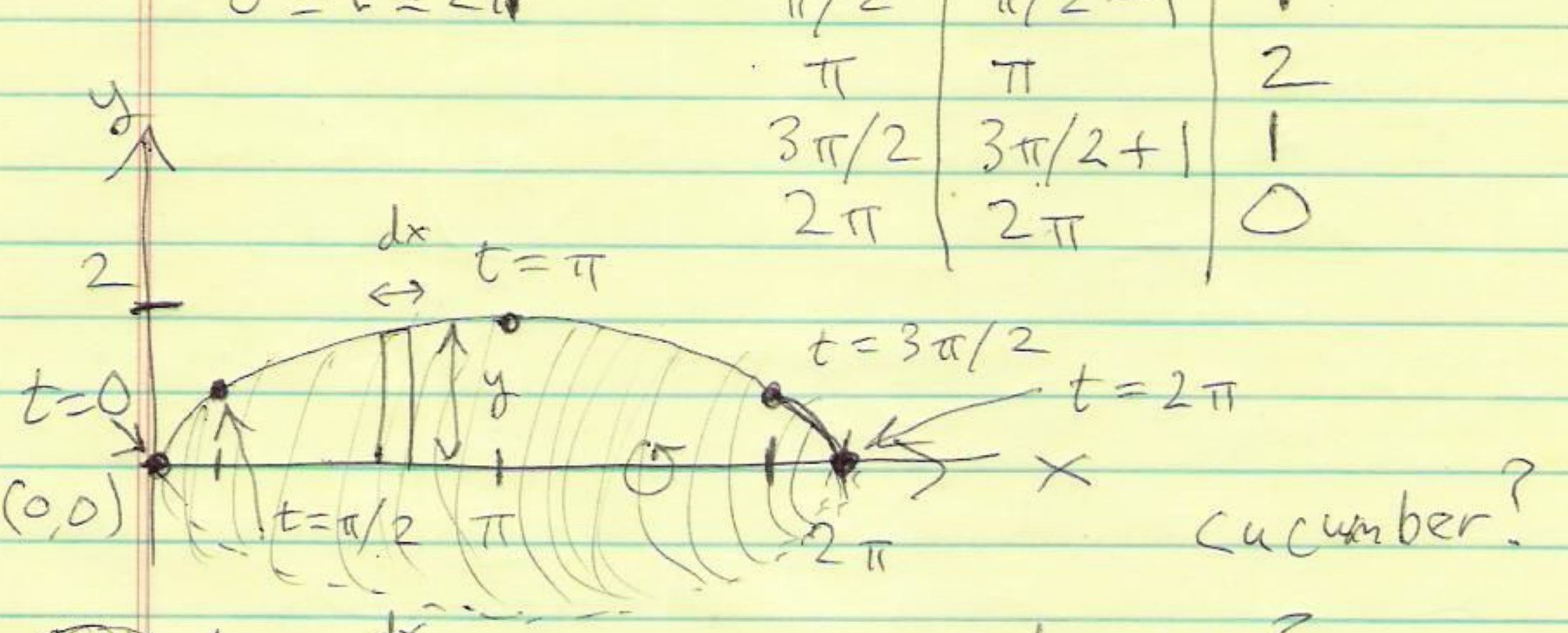
Parametric curves & volume

$x = t - \sin t$

$y = 1 - \cos t$

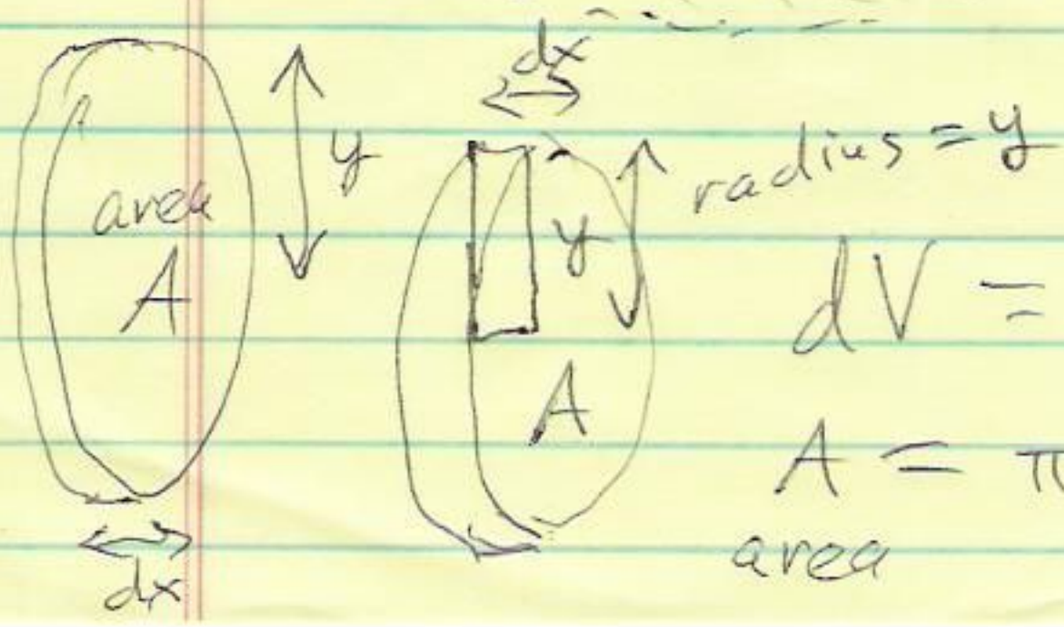
$0 \le t \le 2\pi$

t	x	y
0	0	0
$\pi/2$	$\pi/2 - 1$	1
π	π	2
$3\pi/2$	$3\pi/2 + 1$	1
2π	2π	0



cucumber?

Volume = ?



$dV = A\,dx = \pi y^2 dx$

$A = \pi y^2$ (area)

$V = \int \pi y^2 dx$

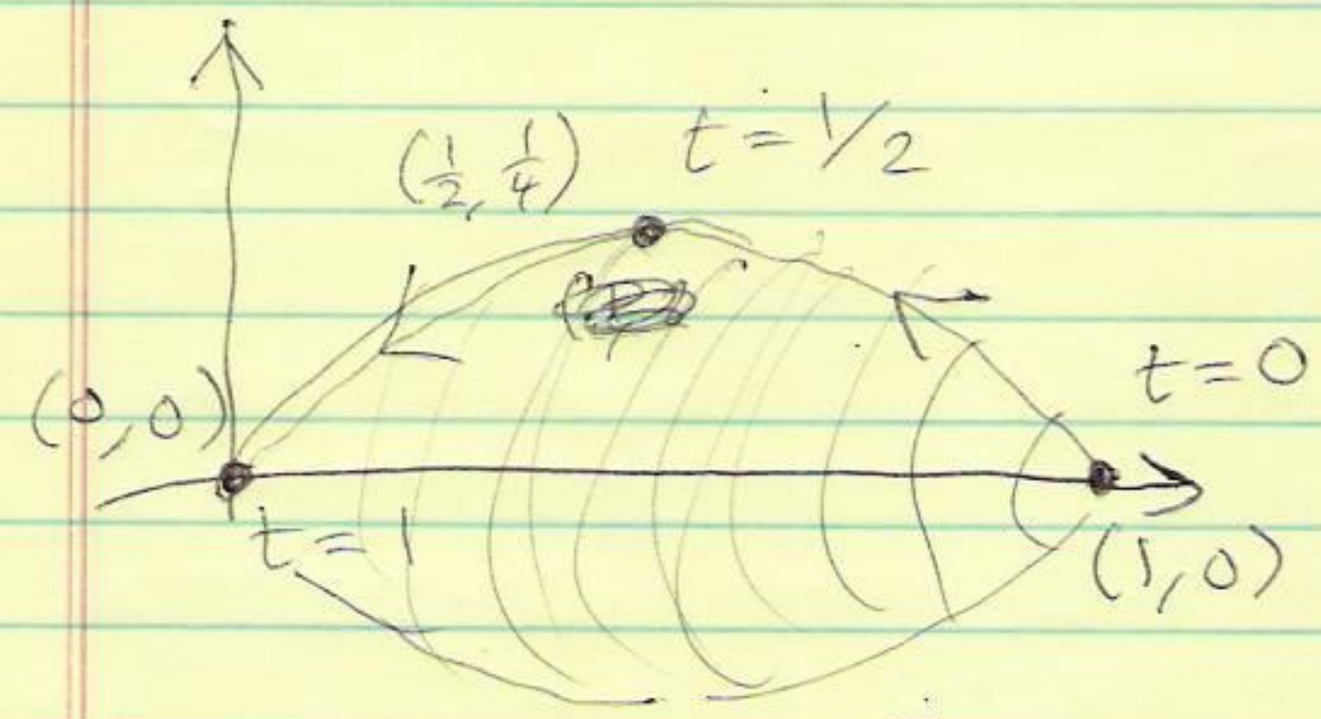
$x = t - \sin t$ $\quad dx = (1-\cos t)\,dt$

$y = 1 - \cos t$ $\quad y^2 = (1-\cos t)^2$

$0 \le t \le 2\pi$

$$V = \int_0^{2\pi} \pi (1-\cos t)^2 (1-\cos t)\,dt$$

$$= \pi \int_0^{2\pi} (1-\cos t)^3\,dt$$



$x = 1 - t$

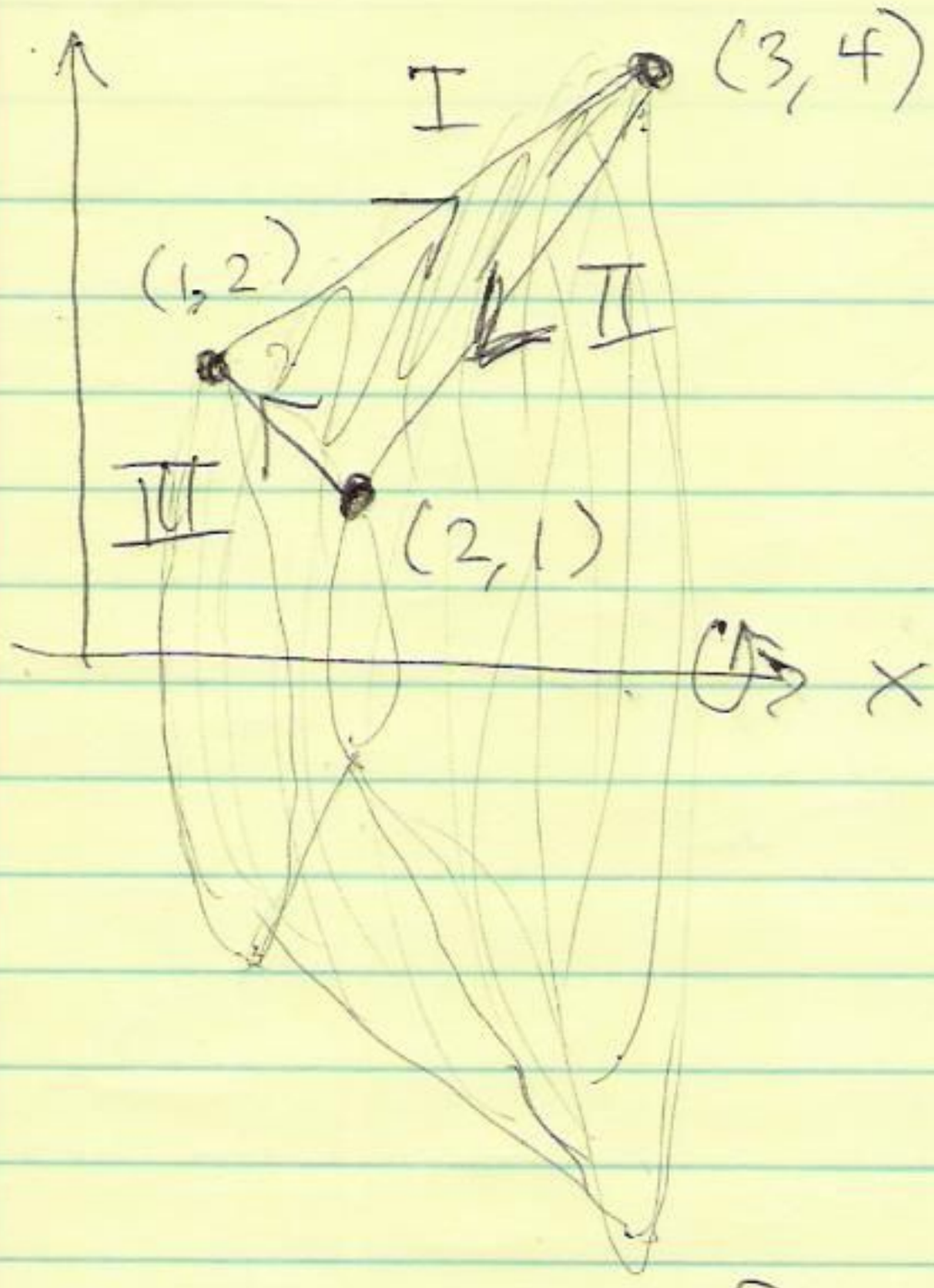
$y = t - t^2$

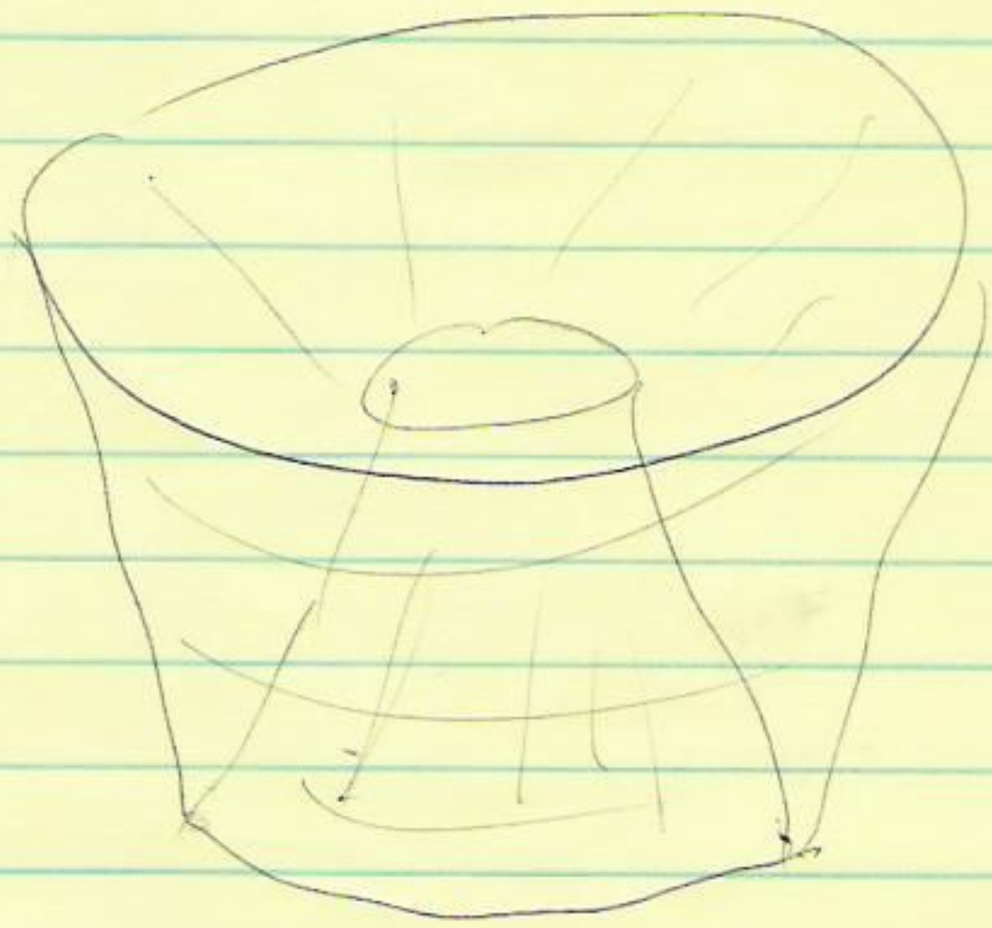
$0 \le t \le 1$

football?

volume = ? ← HW #2

the curve goes to the left* as t increases, which makes $dx < 0$. So, use $dV = -\pi y^2 dx$.

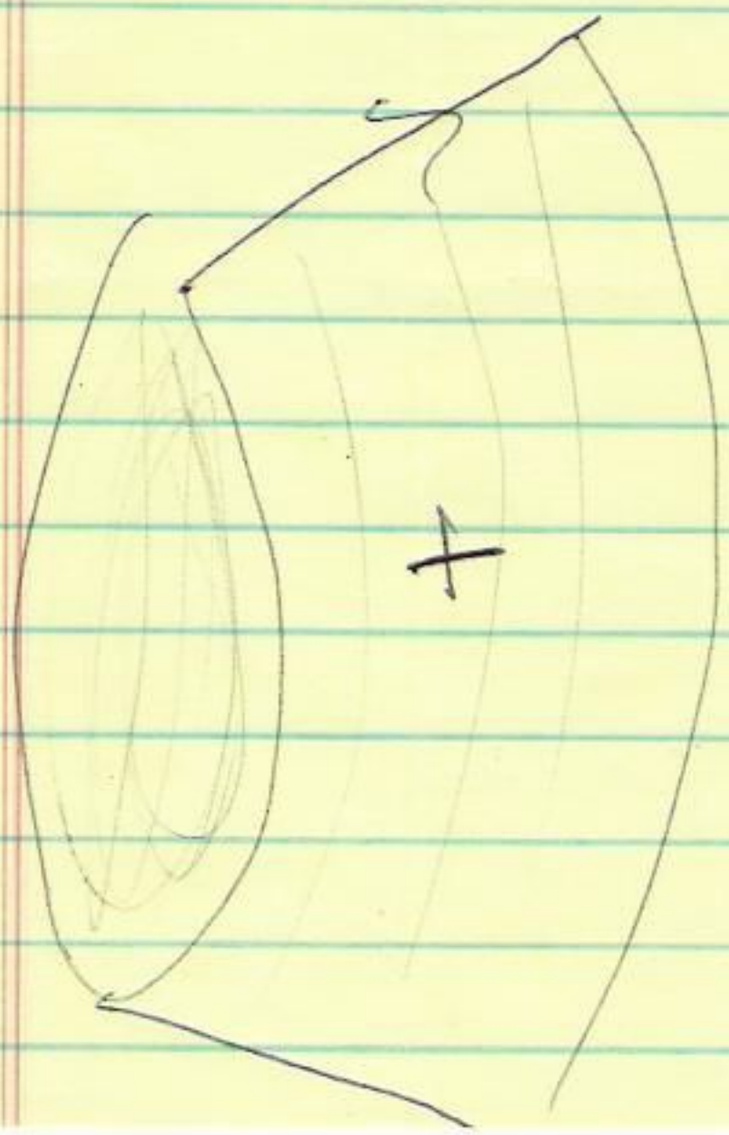




Claim: $V = \int_{\text{loop}} \pi y^2 \, dx$

(clockwise)

I : $dx > 0$ II : $dx < 0$ III : $dx < 0$



$\int_I \pi y^2 dx = ?$

$x = 1+2t$

$y = 2+2t$

$dx = 2\,dt$

$x = 1(1-t)+3t$

$y = 2(1-t)+4t$

start: $t=0$

end: $t=1$

$y^2 = (2+2t)^2$

$$\int_I \pi y^2 dx = \int_0^1 \pi (2+2t)^2 2\,dt$$

$$= \int_0^1 \pi (4+8t+4t^2) 2\,dt$$

$$= 2\pi \int_0^1 (4+8t+4t^2)\,dt$$

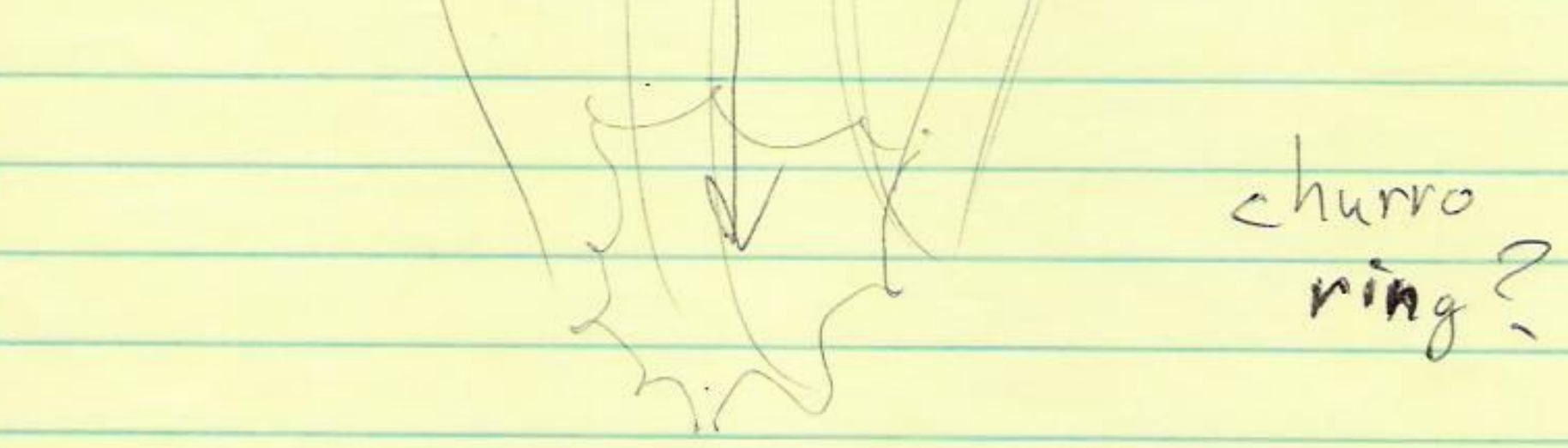
$$= 2\pi \left(4t+4t^2+\frac{4}{3}t^3\right)\Big|_0^1$$

$$= 2\pi\left(4+4+\frac{4}{3}\right) = 56\pi/3$$

$$V = \underbrace{\int_I \pi y^2 dx}_{56\pi/3} + \underbrace{\int_{II} \pi y^2 dx + \int_{III} \pi y^2 dx}_{\text{negative}}$$

$$x = 10\cos(t) + \sin(8t)$$
$$y = 10\sin(t) + \cos(8t) + 12$$
$$0 \le t \le 2\pi$$

HW #3
Express the volume as an integral involving only t.



churro ring?

If the loop is clockwise, then

$$V = \int_{loop} \pi y^2 dx$$

If the loop is counterclockwise, then

$$V = -\int_{loop} \pi y^2 dx$$