HW46 Assuming density 1, mass $= M = \int_0^{4\pi} \sqrt{\underbrace{(2t)^2}_{dx/dt} + \underbrace{(-2\sin(2t))^2}_{dy/dt} + \underbrace{(2\cos(t))^2}_{dz/dt}}\, dt = 161.643\ldots$

$$x_{cm} = \frac{1}{M}\int_0^{4\pi} \underbrace{t^2}_{x} \underbrace{\sqrt{(2t)^2 + \cdots}\, dt}_{1\, ds = dm} = 77.617\ldots$$

$$y_{cm} = \frac{1}{M}\int_0^{4\pi} \underbrace{\cos(2t)}_{y} \underbrace{\sqrt{(2t)^2 + \cdots}\, dt}_{1\, ds = dm} = 0.0071488\ldots$$

$$z_{cm} = \frac{1}{M}\int_0^{4\pi} \underbrace{2\sin(t)}_{z} \underbrace{\sqrt{(2t)^2 + \cdots}\, dt}_{1\, ds = dm} = -0.30015\ldots$$

HW47 $\vec{F}\cdot d\vec{r} = y\,dx - x\,dy$; $\vec{G}\cdot d\vec{r} = xy\,dx - 2\,dy$

I: $x = 3\cos(t)$, $y = 3\sin(t)$, $0 \le t \le 2\pi$

II: $x = 3+5t$, $y = 2t$, $0 \le t \le 1$

III: $x = 8-5t$, $y = 2+2t$, $0 \le t \le 1$

IV: $x = 3$, $y = 4t$, $0 \le t \le 1$

$$\int_{I} \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \left[(3\sin t)(-3\sin t\,dt) - (3\cos t)(3\cos t\,dt)\right]$$

$$= \int_0^{2\pi} -9\,dt = \boxed{-18\pi}$$

$$\int_{II+III} \vec{F}\cdot d\vec{r} = \int_0^1 \left[(2t)(5\,dt) - (3+5t)(2\,dt)\right]$$

$$+ \int_0^1 \left[(2+2t)(-5\,dt) - (8-5t)(2\,dt)\right]$$

$$= \int_0^1 -6\,dt + \int_0^1 -26\,dt = \boxed{-32}$$

$$\int_{IV} \vec{F}\cdot d\vec{r} = \int_0^1 \left[(4t)(0\,dt) - (3)(4\,dt)\right] = \boxed{-12}$$

$$\int_{I} \vec{G}\cdot d\vec{r} = \int_0^{2\pi} \left[(3\cos t)(3\sin t)(-3\sin t\, dt) - 2(3\cos t\, dt)\right]$$

$$= \boxed{0}$$

$$\int_{II+III} \vec{G}\cdot d\vec{r} = \int_0^1 \left[(3+5t)(2t)(5\,dt) - 2(2\,dt)\right]$$

$$+ \int_0^1 \left[(8-5t)(2+2t)(-5\,dt) - 2(2\,dt)\right]$$

$$= 83/3 - 247/3 = \boxed{-164/3}$$

$$\int_{IV} \vec{G}\cdot d\vec{r} = \int_0^1 \left[(3)(4t)(0\,dt) - 2(4\,dt)\right]$$

$$= \int_0^1 -8\,dt = \boxed{-8}$$

(2) $f(x,y) = \int P\,dx = \int \frac{x\,dx}{(x^2+y^2)^{3/2}} = \int \frac{du/2}{u^{3/2}} = \frac{1}{2} \cdot \frac{u^{-1/2}}{-1/2} + g(y)$

$u = x^2+y^2;\ du = 2x\,dx$

$\left[\int u^{-\frac{3}{2}}\,du = \frac{u^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}\right]$

$= \frac{-1}{\sqrt{x^2+y^2}} + g(y)$ $\quad \left[d(u^{-1/2}) = -\frac{1}{2} u^{-\frac{1}{2}-1}\,du\right]$

$f_y = \frac{(-1)(-1/2)(2y)}{(x^2+y^2)^{3/2}} + g'(y) = \frac{y}{(x^2+y^2)^{3/2}} = Q$

$g'(y) = 0 \Rightarrow g(y) =$ constant; we choose 0.

$f(x,y) = \boxed{-1/\sqrt{x^2+y^2}}$

$\int_D \vec{\nabla} f \cdot d\vec{r} = f(3,4) - f(1,1) = \boxed{\frac{-1}{5} - \frac{-1}{\sqrt{2}}}$

$= \frac{1}{\sqrt{2}} - \frac{1}{5} = 0.259\ldots$

HW49.

(1) $f(x,y,z) = \int \overbrace{\sin(y)}^{P}\,dx = x\sin(y) + g(y,z)$

$f_y = x\cos(y) + g_y = x\cos(y) + \cos(z) = Q$

$g(y,z) = \int \cos(z)\,dy = y\cos(z) + h(z)$

$f_z = 0 + g_z = -y\sin(z) + h'(z) = -y\sin z = R$

$h'(z) = 0 \Rightarrow h(z) = \text{constant};$ choose constant 0.

$f(x,y,z) = \boxed{x\sin(y) + y\cos(z) + 0}$

$\int_C \vec{\nabla} f \cdot d\vec{r} = f(\pi,\pi,\pi) - f(0,0,0) = \boxed{-\pi}$

HW50 $\iint_D x\, dA = \int_{\partial D} \frac{x^2}{2}\, dy$

$\iint_D y\, dA = \int_{\partial D} -\frac{y^2}{2}\, dx$ $\Leftarrow$ 4 instances of Green's Theorem:

$A = \iint_D 1\, dA = \int_{\partial D} x\, dy$

$\iint_D (x^2+y^2)\, dA = \int_{\partial D} \frac{x^3 dy - y^3 dx}{3}$

P	Q	$Q_x - P_y$
0	$x^2/2$	$x - 0$
$-y^2/2$	0	$0-(-y)$
0	x	$1 - 0$
$-y^3/3$	$x^3/3$	$x^2-(-y^2)$

$\int_{\partial D} \frac{x^2}{2}\, dy = \int_0^{2\pi} \left[(2+\cos(7t))\cos(t)\right]^2 \left(\frac{1}{2}\right)\left[-7\sin(7t)\sin(t) + (3+\cos(7t))\cos(t)\right] dt = 0$

Similarly, $\int_{\partial D} \frac{y^2}{2}\, dx = \int_0^{2\pi} \ldots = 0$ & $\int_{\partial D} \frac{x^3 dy - y^3 dx}{3} = \int_0^{2\pi} \ldots \approx 91.3$.

So, $x_{cm} = y_{cm} = \boxed{0}$ and average$(x^2+y^2) \approx \frac{91.3}{A} \approx \boxed{4.47}$