HW 34

(1) Find the center of mass of the region $\{(x,y) \mid x,y \geq 0 \ \& \ 4 \leq x^2+y^2 \leq 9\}$.

(2) Check out this trick: An important integral

From statistics, $\int_{-\infty}^{\infty} e^{-x^2/2}\,dx$, equals $\sqrt{\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\,dx \int_{-\infty}^{\infty} e^{-\frac{y^2}{2}}\,dy}$

$$= \sqrt{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\,dx\,dy} = \sqrt{\int_0^{\infty}\int_0^{2\pi} e^{-r^2/2}\,r\,d\theta\,dr}$$

$$= \sqrt{2\pi \int_0^{-\infty} e^u(-du)} = \sqrt{2\pi(-(0-1))} = \sqrt{2\pi}$$

Use a similar trick to evaluate $\int_0^{\infty}\int_0^{\infty} \frac{dx\,dy}{(x^2+y^2)^2+1}$. You don't need the $\sqrt{\ }$ part of the trick. But polar coordinates will really help!

(3) Find $\iint_S xy\,dx\,dy$ where $S = \{(x,y) \mid r \leq \theta \leq \pi\}$.