HW 47

(1) Let $\langle P, Q \rangle = \langle 6x^2 + 10xy^4 - 3, 20x^2y^3 + 1 \rangle$.

Find the general solution to $\vec{\nabla} f = \langle P, Q \rangle$.

(2) Find f as in (1) such that $f(1,1) = 7$.

(3) Find $f(x,y)$ such that $f(1,0) = 1$ and

$$\vec{\nabla} f(x,y) = \left\langle \frac{-y^2}{x^2\sqrt{x^2+y^2}}, \frac{y}{x\sqrt{x^2+y^2}} \right\rangle$$

for all $(x,y) \in D = \{(x,y) \in \mathbb{R}^2 \mid x > 0\}$.

(4) Repeat (3) with $f(-1,0) = 5$ and $D = \{(x,y) \in \mathbb{R}^2 \mid x < 0\}$ (but same $\vec{\nabla} f$ formula).

(5) Does $\dfrac{\langle x, y \rangle}{(x^2+y^2)^{3/2}}$ equal some $\vec{\nabla} f$ on all of $D = \mathbb{R}^2 - \{(0,0)\}$ (which is not simply connected)?