Ramiro Castill
4/27/16
MATH 3310

HW 22

(a) $\underset{A}{\begin{bmatrix} 3 & 4 \\ 7 & 7 \\ 1 & 2 \end{bmatrix}}$ $\left[\begin{array}{cc|ccc} 3 & 4 & 1 & 0 & 0 \\ 7 & 7 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 & 1 \end{array}\right] \xmapsto{RREF} \begin{array}{c} \text{(1) (2)} = n \leq m \\ \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 2/7 & -1 \\ 0 & 1 & 0 & -1/7 & 1 \\ 0 & 0 & 1 & -2/7 & -1 \end{array}\right] \\ L \quad\quad R \end{array}$ $\underbrace{L^T R A}_{\text{left inverse}} = I_n$, $L^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$L^T * R = \begin{bmatrix} 0 & 2/7 & -1 \\ 0 & -1/2 & 1 \end{bmatrix}$ ← The left inverse of matrix A

rank(A) = m ≤ n (right inverse)

(b) $\begin{bmatrix} 3 & 7 & 6 & 1 & 0 & 5 \\ 3 & 0 & 6 & 0 & 0 & 7 \\ 0 & 1 & 4 & 0 & 0 & 8 \end{bmatrix}$ $\left[\begin{array}{cccccc|ccc} 3 & 7 & 6 & 1 & 0 & 5 & 1 & 0 & 0 \\ 3 & 0 & 6 & 0 & 0 & 7 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 8 & 0 & 0 & 1 \end{array}\right] \xrightarrow{RREF} \begin{array}{c} \text{(1) (1) (3)} \\ \left[\begin{array}{cccccc|ccc} 1 & 0 & 0 & \frac{1}{14} & 0 & -\frac{38}{21} & \frac{1}{14} & \frac{11}{42} & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{7} & 0 & -\frac{2}{7} & \frac{1}{7} & -\frac{1}{7} & 0 \\ 0 & 0 & 1 & -\frac{1}{28} & 0 & \frac{29}{14} & -\frac{1}{28} & \frac{1}{28} & \frac{1}{4} \end{array}\right] \\ L \quad\quad\quad\quad R \end{array}$

$P^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

$AP^T R = I_m$

$P^T R = \begin{bmatrix} 1/14 & 11/42 & -1/2 \\ 1/7 & -1/7 & 0 \\ -1/28 & 1/28 & 1/4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (Right inverse)

rank(A) = m = n

(c) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ $\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \xmapsto{RREF} \begin{array}{c} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right] \\ L \quad\quad R = A^{-1} \end{array}$

$A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

rank(A) = m = n

(d) $\begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$ $\left[\begin{array}{cc|cc} 7 & 2 & 1 & 0 \\ 1 & 6 & 0 & 1 \end{array}\right] \xmapsto{RREF} \begin{array}{c} \left[\begin{array}{cc|cc} 1 & 0 & 3/20 & -1/20 \\ 0 & 1 & -1/40 & 7/40 \end{array}\right] \\ L \quad\quad R = A^{-1} \end{array}$

$A^{-1} = \begin{bmatrix} 3/20 & -1/20 \\ -1/40 & 7/40 \end{bmatrix}$

rank(A) = n ≤ m (left inv.)

(e) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $\left[\begin{array}{c|cc} 4 & 1 & 0 \\ 3 & 0 & 1 \end{array}\right] \xmapsto{RREF} \begin{array}{c} \left[\begin{array}{c|cc} 1 & 0 & 1/3 \\ 0 & 1 & -4/3 \end{array}\right] \\ L \quad R \end{array}$ $\underbrace{L^T R A}_{\text{left inverse}} = I_n$ $L^T = \begin{bmatrix} 1 & 0 \end{bmatrix}$

$L^T R = \begin{bmatrix} 0 & 1/3 \end{bmatrix}$ (left inverse)

rank(A) = m ≤ n (right)

(f) $\begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$ $\left[\begin{array}{ccc|c} 3 & 4 & 5 & 1 \end{array}\right] \mapsto \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 4/3 & 5/3 & 1/3 \end{array}\right] \\ L \quad\quad R \end{array}$ $AP^T R = I_m$

$P = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ $P^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $P^T R$ = right inverse = $\begin{bmatrix} 1/3 \\ 0 \\ 0 \end{bmatrix}$

HW22

Besides my methods "L^TR" & "P^TR" for finding left & right inverses, some of you looked up another valid method: $(A^TA)^{-1}A^T$ & $A^T(AA^T)^{-1}$. (If A is not invertible, but still has a left or right inverse, then that one-sided inverse is not unique. A left inverse B of A can be verified by checking $BA=I$; check $AB=I$ for right inverses.)

$\vec{a} = -\vec{e}_1 + \vec{e}_2 + \vec{e}_3 = \vec{u}_1 \qquad \vec{u}_2 = \vec{e}_1 \qquad \vec{u}_3 = \vec{e}_3$

Orthogonal basis:

$$\vec{v}_1 = \vec{u}_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{v}_2 = \vec{u}_2 - proj_{\vec{v}_1}\vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \frac{\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}\cdot\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}{\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}\cdot\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}}\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 2/3 \\ -1/3 \end{bmatrix}$$

$1/3$

$$\vec{V}_3 = \vec{u}_3 - proj_{\vec{v}_1}\vec{u}_3 - proj_{\vec{v}_2}\vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \frac{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\cdot\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}\cdot\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}}\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} - \frac{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\cdot\begin{bmatrix} 1/3 \\ 2/3 \\ -1/3 \end{bmatrix}}{\begin{bmatrix} 1/3 \\ 2/3 \\ -1/3 \end{bmatrix}\cdot\begin{bmatrix} 1/3 \\ 2/3 \\ -1/3 \end{bmatrix}}\begin{bmatrix} 1/3 \\ 2/3 \\ -1/3 \end{bmatrix} \quad \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}$$

$1/3$ $\qquad \frac{-1/3}{2/3} = -\frac{1}{2}$ $\qquad 1/9 + 4/9 + 1/9 = 2/3$

$$\vec{V}_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = -\vec{e}_1 + \vec{e}_2 + \vec{e}_3 \; ; \; \vec{V}_2 = \begin{bmatrix} 1/3 \\ 2/3 \\ -1/3 \end{bmatrix} ; \; \vec{V}_3 = \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}$$

Orthonormal basis:

$$\vec{W}_1 = \frac{\vec{V}_1}{\sqrt{\vec{V}_1\cdot\vec{V}_1}} = \frac{1}{\sqrt{3}}\vec{V}_1 = \begin{bmatrix} -1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} \qquad \vec{W}_2 = \frac{\vec{V}_2}{\sqrt{\vec{V}_2\cdot\vec{V}_2}} = \frac{\sqrt{3}}{\sqrt{2}}\vec{V}_2 = \begin{bmatrix} \sqrt{3}/3\sqrt{2} \\ 2\sqrt{3}/3\sqrt{2} \\ -\sqrt{3}/3\sqrt{2} \end{bmatrix}$$

$$\vec{W}_3 = \frac{\vec{V}_3}{\sqrt{\vec{V}_3\cdot\vec{V}_3}} = \frac{1}{\sqrt{2}}\vec{V}_3 = \begin{bmatrix} 1/2\sqrt{2} \\ 0 \\ 1/2\sqrt{2} \end{bmatrix}$$

K = matrix of $T = AHA^{-1}$

$$A = \begin{bmatrix} -\sqrt{3}/3 & \sqrt{6}/6 & \sqrt{2}/4 \\ \sqrt{3}/3 & \sqrt{6}/3 & 0 \\ \sqrt{3}/3 & -\sqrt{6}/6 & \sqrt{2}/4 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} -\sqrt{3}/3 & \sqrt{3}/3 & \sqrt{3}/3 \\ \sqrt{6}/6 & \sqrt{6}/3 & -\sqrt{6}/6 \\ \sqrt{2} & 0 & \sqrt{2} \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(30°) & -\sin(30°) \\ 0 & \sin(30°) & \cos(30°) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$K = \begin{bmatrix} \frac{5\sqrt{3}}{24} + \frac{1}{3} & \frac{\sqrt{3}}{4} - \frac{1}{3} & -\frac{\sqrt{3}}{24} - \frac{1}{3} \\ -\frac{\sqrt{3}}{6} - \frac{1}{3} & \frac{\sqrt{3}}{3} + \frac{1}{3} & \frac{1}{3} \;\; \frac{\sqrt{3}}{2} \\ \frac{3\sqrt{3}}{8} - \frac{1}{3} & \frac{1}{3} - \frac{\sqrt{3}}{12} & \frac{11\sqrt{3}}{24} + \frac{1}{3} \end{bmatrix}$$

HW 24

$$\vec{w}_1 = \begin{bmatrix}1\\0\\0\\0\end{bmatrix},\quad \vec{w}_2 = \begin{bmatrix}1\\1\\4\\4\end{bmatrix},\quad \vec{w}_3 = \begin{bmatrix}3\\2\\1\\0\end{bmatrix},\quad \vec{w}_4 = \begin{bmatrix}0\\0\\1\\9\end{bmatrix}$$

$$\vec{v}_1 = \vec{w}_1 \;;\quad \vec{v}_2 = \begin{bmatrix}1\\1\\4\\4\end{bmatrix} - \frac{\begin{bmatrix}1\\1\\4\\4\end{bmatrix}\cdot\begin{bmatrix}1\\0\\0\\0\end{bmatrix}}{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\cdot\begin{bmatrix}1\\0\\0\\0\end{bmatrix}}\begin{bmatrix}1\\0\\0\\0\end{bmatrix} = \begin{bmatrix}0\\1\\4\\4\end{bmatrix}$$

$$\vec{v}_3 = \begin{bmatrix}3\\2\\1\\0\end{bmatrix} - \frac{\begin{bmatrix}3\\2\\1\\0\end{bmatrix}\cdot\begin{bmatrix}1\\0\\0\\0\end{bmatrix}}{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\cdot\begin{bmatrix}1\\0\\0\\0\end{bmatrix}}^{3}\begin{bmatrix}1\\0\\0\\0\end{bmatrix} - \frac{\begin{bmatrix}3\\2\\1\\0\end{bmatrix}\cdot\begin{bmatrix}0\\1\\4\\4\end{bmatrix}}{\begin{bmatrix}0\\1\\4\\4\end{bmatrix}\cdot\begin{bmatrix}0\\1\\4\\4\end{bmatrix}}^{6/33}\begin{bmatrix}0\\1\\4\\4\end{bmatrix} = \begin{bmatrix}0\\60/33\\3/11\\-24/33\end{bmatrix}$$

$$\vec{v}_4 = \begin{bmatrix}0\\0\\1\\9\end{bmatrix} - 0 - \frac{\begin{bmatrix}0\\0\\1\\9\end{bmatrix}\cdot\begin{bmatrix}0\\1\\4\\4\end{bmatrix}}{\begin{bmatrix}0\\1\\4\\4\end{bmatrix}\cdot\begin{bmatrix}0\\1\\4\\4\end{bmatrix}}^{40/33}\begin{bmatrix}0\\1\\4\\4\end{bmatrix} - \frac{\begin{bmatrix}0\\0\\1\\9\end{bmatrix}\cdot\begin{bmatrix}0\\60/33\\3/11\\-24/33\end{bmatrix}}{\begin{bmatrix}0\\60/33\\3/11\\-24/33\end{bmatrix}\cdot\begin{bmatrix}0\\60/33\\3/11\\-24/33\end{bmatrix}}^{-69/11 \,/\, 43/11 \,=\, -69/43}\begin{bmatrix}0\\60/33\\3/11\\-24/33\end{bmatrix} = \begin{bmatrix}0\\220/129\\-440/129\\385/129\end{bmatrix}$$

$$\hat{u}_1 = \frac{1}{\sqrt{1}}\begin{bmatrix}1\\0\\0\\0\end{bmatrix} \qquad \hat{u}_2 = \frac{1}{\sqrt{33}}\begin{bmatrix}0\\1\\4\\4\end{bmatrix} = \begin{bmatrix}0\\\sqrt{33}/33\\4\sqrt{33}/33\\4\sqrt{33}/33\end{bmatrix}$$

$$\hat{u}_3 = \frac{1}{\sqrt{\frac{43}{11}}}\begin{bmatrix}0\\60/33\\3/11\\-24/33\end{bmatrix} = \begin{bmatrix}0\\20\sqrt{473}/473\\3\sqrt{473}/473\\-8\sqrt{473}/473\end{bmatrix}$$

$$\hat{u}_4 = \frac{\sqrt{129}}{55}\begin{bmatrix}0\\220/129\\-440/129\\385/129\end{bmatrix} = \begin{bmatrix}0\\4\sqrt{129}/129\\-8\sqrt{129}/129\\7\sqrt{129}/129\end{bmatrix}$$

$$A = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \sqrt{33}/33 & 20\sqrt{473}/473 & 4\sqrt{129}/129\\ 0 & 4\sqrt{33}/33 & 3\sqrt{473}/473 & -8\sqrt{129}/129\\ 0 & 4\sqrt{33}/33 & -8\sqrt{473}/473 & 7\sqrt{129}/129\end{bmatrix}$$

$$A^T = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \sqrt{33}/33 & 4\sqrt{33}/33 & 4\sqrt{33}/33\\ 0 & 20\sqrt{473}/473 & 3\sqrt{473}/473 & -8\sqrt{473}/473\\ 0 & 4\sqrt{129}/129 & -8\sqrt{129}/129 & 7\sqrt{129}/129\end{bmatrix} = A^{-1}$$

$A^T A = I_4$ ✓

$A A^T = I_4$ ✓

HW 25

(1)
$$A = \begin{array}{ccc} \vec{1} & \vec{Y} & \vec{Y}^2 \end{array}$$
$$A = \begin{bmatrix} 1 & 1790 & 1790^2 \\ 1 & 1800 & 1800^2 \\ 1 & 1810 & 1810^2 \\ 1 & 1820 & 1820^2 \\ \vdots & \vdots & \vdots \\ 1 & 2010 & 2010^2 \end{bmatrix} \xrightarrow{QR} \overset{Q}{\begin{bmatrix} \frac{\sqrt{23}}{23} & -\frac{\sqrt{253}}{46} & \frac{\sqrt{8855}}{230} \\ \vdots & -\frac{5\sqrt{253}}{253} & \frac{4\sqrt{8855}}{1265} \\ \vdots & \vdots & \vdots \\ \vdots & \frac{\sqrt{253}}{46} & \frac{\sqrt{8855}}{230} \end{bmatrix}} \begin{bmatrix} \sqrt{23} & 1900\sqrt{23} & 3614400\sqrt{23} \\ 0 & 20\sqrt{253} & 76000\sqrt{253} \\ 0 & 0 & 200\sqrt{8855} \end{bmatrix}$$

$$P = \begin{bmatrix} 3929214 \\ 5236631 \\ \vdots \\ \vdots \\ 308745538 \end{bmatrix} \qquad B = Q^T \cdot P$$

$$C = R^{-1} \cdot B = \begin{bmatrix} 2.20 \cdot 10^{10} \\ -2.44 \times 10^{7} \\ 6.78 \cdot 10^{3} \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix} \Big\} \; a\vec{1} + b\vec{Y} + c\vec{Y}^2$$

$$D = \begin{bmatrix} \vec{1} & \vec{Y} \end{bmatrix} \xrightarrow{QR} \overset{Q}{\begin{bmatrix} \frac{\sqrt{23}}{23} & -\frac{\sqrt{253}}{46} \\ \vdots & -\frac{5\sqrt{253}}{253} \\ \vdots & \vdots \end{bmatrix}}, \overset{R}{\begin{bmatrix} \sqrt{23} & 1900\sqrt{23} \\ 0 & 20\sqrt{253} \end{bmatrix}} \qquad E = Q^T \cdot P_2$$

$$F = R^{-1} \cdot E = \begin{bmatrix} -19.5239 \\ 0.019652 \end{bmatrix} \begin{matrix} d \\ e \end{matrix}$$

$$\ln(P) = P_2 = \begin{bmatrix} 15.184 \\ \vdots \\ 19.977 \end{bmatrix} \qquad d\vec{1} + e\vec{Y}$$

(2)
$$U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{QR} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = V$$
$$\begin{array}{ccc} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{array}$$

(3) M of T for $\vec{e}_1, \vec{e}_2, \vec{e}_3$

$$M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

(5)
$$M^T = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} = M$$

(4)
$$\vec{w}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \vec{w}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \vec{w}_3 = \begin{bmatrix} 2 \\ 16 \\ -2 \end{bmatrix} \quad \vec{w}_4 = \begin{bmatrix} 6 \\ -2 \\ 0 \end{bmatrix} \quad \vec{w}_5 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\vec{w}_1 \cdot \vec{w}_1 M = 7 > \vec{0}$

$\vec{w}_2 \cdot \vec{w}_2 M = 8 > \vec{0}$

$\vec{w}_3 \cdot \vec{w}_3 M = 60 > \vec{0}$

$\vec{w}_4 \cdot \vec{w}_4 M = 10 > \vec{0}$

$\vec{w}_5 \cdot \vec{w}_5 M = 12 > \vec{0}$