Day 17

① To "rescale" by a factor of 3 in the x-direction & a factor of 4 in the y-direction, in 2D space ($\mathbb{C}^2$), use $A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$. $\begin{bmatrix} x \\ y \end{bmatrix}$ is transformed to $A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x \\ 4y \end{bmatrix}$.

Find the matrix B that rescales by a factor of 3 in the $\vec{u}$-direction and 4 in the $\vec{v}$-direction, where $\vec{u}$ & $\vec{v}$ are $\vec{e}_1$ & $\vec{e}_2$ rotated 30° counterclockwise.

Day 17

(2) Suppose that D is a 6×7 matrix and

$$D \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & -1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\dim(C(D)) =$ ____. $\dim(N(D)) =$ ____.

$\dim(R(D)) =$ ____. $\dim(L(D)) =$ ____.

(3) True/False: If E is a square matrix, then $\dim(N(E)) = \dim(L(E))$. If true, explain why. If False, give a counterexample.