

## MATH 4335 Homework 25

Assume that we have functions  $f, f_1, f_2, f_3, f_4, \dots : [a, b] \rightarrow \mathbb{R}$  such that  $f_1, f_2, f_3, f_4, \dots$  converges uniformly to  $f$ . Further assume that  $\int_a^b f_n(x) dx$  exists for each  $n$ .

Define  $g, g_1, g_2, g_3, g_4, \dots : [a, b] \rightarrow \mathbb{R}$  by  $g(x) = \int_a^x f(t) dt$  and  $g_n(x) = \int_a^x f_n(t) dt$ . By the last claim of the Day 25 lecture, we know that, for each  $x \in [a, b]$ ,  $g(x)$  exists and equals  $\lim(g_n(x))_{n \in \mathbb{N}}$ .

Thus,  $g_1, g_2, g_3, g_4, \dots$  converges pointwise to  $g$ .

Prove that  $g_1, g_2, g_3, g_4, \dots$  also converges uniformly to  $g$ .