MATH 4335 TEST 1

Name:

1.

$$\frac{n-4}{2n+1} \underset{1/50}{\approx} \frac{1}{2} \text{ for } n \gg 1.$$

What does the above statement mean, exactly? Give a precise explanation. Try to explain with words instead of with more symbols.

2. Prove that if a sequence $\{a_n\}$, $n \geq 0$, is increasing for n large, then $\{a_n\}$ is bounded below.

3. Use the binomial theorem to prove that
$$\left(1 + \frac{1}{\sqrt{n}}\right)^n \to \infty.$$

4. Consider the sequence recursively defined by $a_0 = 0$ and $a_{n+1} = \sqrt{3 + 2a_n}$ for $n \geq 0$.

(a) If the sequence does have a limit L, then what must L be?

(b) Prove that $\{a_n\}$ is increasing and bounded above by the number L you found in part (a). (Hint: Prove $a_n < a_{n+1} < L$ by induction.)