MATH 4335 MIDTERM EXAM

Name:

Instructions: Solve two of the four problems. I will grade the first two problems for which I see a proof attempt that has not been crossed out.

Warning: For each problem, I only want your single best proof attempt, with any other proof attempts crossed out. If you do not heed my warning, I will grade based on the worst proof attempt.

Date: Oct. 10, 2016.

Exercise	Grade
1	
2	
3	
4	
Overall	

1. Let $a_0 = 1/2$ and $a_{n+1} = a_n^3$ for all $n \geq 0$. Prove by induction that $\{a_n\}$ is a bounded and monotone sequence.

2. Prove $\frac{5}{n^3} - \frac{1}{n^2} \to 0$ directly from the definition of limit.

3. Prove that the sequence $\left\{ \frac{1}{\sqrt{0!}} + \frac{1}{\sqrt{1!}} + \frac{1}{\sqrt{2!}} + \cdots + \frac{1}{\sqrt{n!}} \right\}$ is bounded above.

4. Prove that $\left(1 + \frac{1}{n^2}\right)^{\left(n^3\right)} \to \infty$. Do not use logarithms.